IMPROVED CUBIC VOLUME PREDICTION FOR INLAND DOUGLAS-FIR USING A NEW MEASURE OF FORM-FACTOR

John L. Mathis and Krishna P. Rustagi

ABSTRACT

Accuracy of upper stem (above breast-height) form-factor can be significantly increased by using a new variable — the ratio of height at two-thirds breast-height diameter to total tree height ($R_{.67Da}$) — both measured above breast-height. This form-factor provides an accurate estimate of upper-stem cubic volume and, in addition, defines a fully compatible upper-stem taper. The form-factor based volume and taper models should have broader application and practically bias free predictions even when the model is used beyond its geographic range.

When evaluated on 350 inland Douglas-fir tree data from five Region IV Forests (Forest Service), the mean squared deviation (MSD) was reduced by approximately 58% over the best D and H only based models.

Keywords: Douglas-fir, form-factor, stem form, stem taper, volume determination

INTRODUCTION

Accurate estimates of timber volumes are needed for timber cruising, stand examination, inventories, and planning. Usually tree volumes are estimated from equations or tables based on just two measurements: outside bark measurement at breast-height (D) and total height (H), (Schumacher and Hall 1933; Honer 1964; Bruce and DeMars 1974). Implicit in these models is the assumption that all trees of a given D and H have the same form and, therefore, have the same volume. This is not always true. Individual trees and often whole stands have forms that vary greatly from the average form predicted by equations or tables using only D and H.

Recent developments in volume prediction equations include the use of a third parameter such as height to live crown ratio or crown length (Farrar 1985; Burkhart and Walton 1985; Hann *et al.* 1987). Their results showed only marginal improvement in the prediction accuracy of cubic volume. Rustagi and Loveless (1990) introduced a new variable $H_{.67D}$ (height where diameter outside bark (DOB) equals two-thirds of D), for estimating cubic volume in the stem above breast-height, and the upper-stem taper with a simple and yet fully compatible taper model. With $H_{.67D}$ in conjunction with D and H, they were able to obtain reduction in the mean squared deviation (MSD) for volume prediction by 79% over the best D and H only model for coastal Douglas-fir. This was quite significant, since prior effects using crown ratios, height to live crown, tree age, site index, and H:D ratios were not able to reduce the MSD by more than 15 or 20%.

In this study, we applied $H_{.67D}$ to 350 inland Douglas-fir trees with a less spectacular but an impressive reduction of MSD for volume prediction by 58.2% over the best D and H only model.

VARIABLE DEFINITIONS

Measurements used in this paper are in English units: diameters in inches, heights in feet, and volumes in cubic feet. All regressions are linear. Logarithmic formulas are base 10. Following are the variables used in the models for volumes, form-factors, and taper equations:

D	=	Outside bark diameter at breast-height (4.5 ft)
dbh	=	Diameter inside bark at breast height
$\widehat{dbh}$	=	dbh predicted from DOB
d	=	Diameter inside bark at any height above ground
H	=	Total height of tree from ground to tip
H_a	=	Total height above breast-height (H - 4.5)
h	=	Partial height at any diameter inside bark (measured from ground level)
$H_{.67D}$	=	Height above ground where DOB equals two-thirds of D
$H_{.67Da}$	=	$H_{.67D}$ - 4.5
V_a	=	Volume ft^3 of entire tree above breast-height
V_b	=	Volume ft^3 of tree between breast-height and stump height
F_a	=	Cylindrical form-factor above breast-height
	=	$V_a/(.005454 * dbh^2 * H_a)$
$R_{.67Da}$	=	$H_{.67Da}/H_a$
β	=	The shape parameter for the taper model
	=	$(1 - F_a)/(2 * F_a)$
FI	=	Fit Index, used as an alternative to R^2 when comparing regression models with dependent variables in different forms
	=	$1 - [\Sigma(V_a - \hat{V}_a)^2/(\Sigma(V_a - \bar{V}_a)^2)]$ where: $\hat{V}_a$ = estimated V_a; $\bar{V}_a$ = average V_a
MSD	=	Mean squared deviation of the estimated volume
	=	$\Sigma(V_a - \hat{V}_a)^2/(n - m)$ where: (n - m) = the degrees of freedom; the number of trees less the parameters estimated by V_a or F_a
SE_v	=	Standard error of the estimated volume
	=	$\sqrt{(MSD)}$

Published in proceedings of "Interior Douglas-Fir: The Species and Its Management" held February 27 – March 1, 1990 in Spokane, WA, USA.
Compiled and edited by D.M. Baumgartner and J.E. Lotan, Washington State University Extension, Pullman, WA.
(Bulletin Office, WSU, PO Box 645912, Pullman, WA 99164-5912.) MISC0230. 301 pp.

CONCEPTS

Most volume and taper models consider the entire stem as a single unit (Demaerschalk 1972; Newberry and Burkhart 1986; Burkhart and Walton 1986; Kozak 1988; Newnham 1988). This makes it difficult to fit a taper model which behaves satisfactorily in the butt portion and the middle third of the stem — where most of the volume and value is. Prediction of taper in the stem below breast-height does not serve any practical purpose as the lowest foot or so is lost in the stump, and neither lumber not veneer output is affected by the taper below breast-height. To reduce this problem we follow the procedure of Walters and Hann (1986) and Rustagi and Loveless (1990), and treat the stem as consisting of two parts: above breast-height (upper-stem) and below breast-height (lower-stem), with volume estimated separately in the two sections and fitting the taper model only in the upper stem.

The volume in the upper-stem (V_a) is predicted from estimated inside bark diameter (dbh), total height above breast-height (H_a), and the height to two-thirds D above breast-height ($H_{.67Da}$). For example, a V_a prediction model may be:

[1] $$\hat{V}_a = a + b*\widehat{dbh}^2*H_a + c*\widehat{dbh}^2*H_{.67Da}$$

where a, b and c are regression coefficients.

The volume in the lower stem (V_b) may be computed by either treating it as a cylinder having diameter $\widehat{dbh}$, or predicted from D using regression.

The taper model is fitted only in the upper-stem. The stem profile is derived from the predicted inside bark breast-height diameter ($\widehat{dbh}$), H_a and the shape parameter (β) using the simple relationship:

[2] $$\hat{d} = \widehat{dbh}*[(H - h)/H_a]^{\hat{\beta}}$$

where $\hat{d}$ is the predicted diameter at any height h, and $\hat{\beta}$ is computed from the upper-stem form-factor (F_a).

The underlying theory between the form-factor and ratio of height to fractional basal diameter in simple geometric solids of revolution may be found in Rustagi and Loveless (1990). Approximating the upper-stem of a tree with this simple geometric solid of revolution, their procedure, which we have followed in this study, may be described as follows:

1. The upper-stem taper model is based on the predicted inside bark breast-height diameter ($\widehat{dbh}$), obtained from D using linear or non-linear regression. One advantage of this approach is that stand specific bark-thickness prediction models may be used to reduce model bias, which is generally the problem with D and H based models.

2. Predict the upper-stem volume ($\hat{V}_a$) (see [1] above) or its cylindrical form-factor ($\hat{F}_a$), which has a strong linear relationship with the height-ratio ($R_{.67Da} = H_{.67Da}/H_a$). One simple linear relationship to predict F_a could be:

   [3] $$\hat{F}_a = a + b*R_{.67Da}$$

3. If only V_a is known then the following conversion from V_a to F_a is needed:

   [4] $$F_a = V_a/(.005454*dbh^2*H_a)$$

4. The shape parameter (β) is predicted using the relationship:

   [5] $$\hat{\beta} = (1 - \hat{F}_a)/2\hat{F}_a$$

5. Any upper-stem diameter (inside bark) may be predicted using [2] above. If height (h) for a specified upper-stem inside-bark diameter (d) is needed, the same may be obtained by inverting [2]:

   [6] $$\hat{h} = H - H_a*[d/\widehat{dbh}]^{(1/\beta)}$$

The upper-stem form-factor, used in [4] to determine the value of the shape parameter, may be obtained indirectly from any volume prediction model, or estimated directly from $R_{.67Da}$. However, inclusion of $H_{.67Da}$ in any volume prediction model (with or without H_a), results in substantial reduction in the standard error of volume (SE_v). Regardless of the volume prediction model used, the upper-stem taper model theoretically provides total cubic volume for the upper stem identical to that obtained directly from the volume prediction model.

In this study we report the results of applying this method to inland Douglas-fir, and compare the volume and taper predictions with D and H based models.

DATA

The data are from 435 unforked Douglas-fir trees, obtained by destructive sampling during timber inventories from five U.S. Forest Service Region IV Forests, i.e., Ashley, Challis, Salmon, Targhee, and Wasatch. A sub-sample of 350 trees was used to develop the volume and taper prediction models. The data are summarized in Table 1. The remaining 85 trees were used to test and validate the results.

Table 1.—Summary description of the 350 inland Douglas-fir trees used for volume and taper models.

Tree Attribute	Mean	Standard Deviation	Minimum	Maximum
D (in)	11.3	5.1	5.0	36.3
H (ft)	52.5	16.6	22.7	111.3
dbh (in)	10.0	4.6	4.2	33.7
$H_{.67D}$ (ft)	26.7	8.7	10.8	61.0
V_a (cft)	16.9	24.9	0.8	193.0
F_a	0.44	0.05	0.32	0.63

Published in proceedings of "Interior Douglas-Fir: The Species and Its Management" held February 27 – March 1, 1990 in Spokane, WA, USA.
Compiled and edited by D.M. Baumgartner and J.E. Lotan, Washington State University Extension, Pullman, WA.
(Bulletin Office, WSU, PO Box 645912, Pullman, WA 99164-5912.) MISC0230. 301 pp.

THE VOLUME AND UPPER-STEM PROFILE PREDICTION MODELS

The first step in our analysis was the prediction of dbh from D, as the former cannot easily be measured on standing trees. As the scatter between the observed D and dbh was strongly linear, the following simple linear regression model was found to provide a satisfactory fit:

[8] $\widehat{dbh} = 0.04 + 0.888*D$; SE = 0.346; $R^2 = 0.994$

In Table 2, we list the most promising regression models that were fitted to predict V_a or F_a. The first three volume prediction models are based on D and H_a only. The last three models also utilize $H_{.67Da}$, or $R_{.67Da}$.

Once $\widehat{dbh}$ and $\hat{\beta}$ (through F_a) have been obtained, we can predict d for any given h, or h for any given d, using [2] or [6]. The taper model will always satisfy the conditions that d = 0, when h = H, and d = dbh when h = 4.5. Besides, the volume of the upper-stem obtained from the taper model will be fully compatible with the volume obtained directly from the corresponding volume function.

Depending on the requirements, V_b could be estimated for the entire 4.5 ft length, or only for the section between stump-height and breast-height. We developed a volume prediction model for the 3.5 foot section (above one-foot stump) as follows

[9] $V_b = 0.152 + 0.016938*D^2$; SE = 0.443; $R^2 = 0.976$

RESULTS AND DISCUSSION

The SE_v and Fit Index (FI) values of different regression models (Table 2) clearly establish the important role $H_{.67Da}$ plays in improving the prediction accuracy of the upper-stem volume. Although the reduction in FI is only marginal, the SE_v of the best two variable models is decreased 35% by inclusion of $H_{.67Da}$ as the third independent variable.

Irrespective of whether V_a, $\log(V_a)$ or F_a is used as the dependent variable, inclusion of $H_{.67Da}$ causes sizable reduction in the SE_v. This increased accuracy in predicted volume is carried over to the taper model because of direct relationship between F_a and β. This fact does not hold for taper models in general. The summary results of these regression models on the validation sample are presented in Table 3 which confirms the results of Table 2.

As the taper model is based on a single form parameter, it tends to over-predict the diameters in the uppermost section of the stem. Because very little volume is involved, the taper rate in this part of the stem would have only insignificant impact on the total cubic volume in the upper-stem. Therefore, we consider the stem above a 4 inch dib as a cone regardless of the estimated β coefficient value.

The taper model lacks an inflection point. As such, it cannot model the change in geometric shape from a neiloid to a paraboloid associated with butt swell. This should not cause any problem if the butt-swell does not extend beyond breast-height, or is not very pronounced. If butt-swell extends above breast-height, the diameters in the lower-most part of the stem may also be overpredicted. This overprediction is generally only by a fraction of an inch and is well within the prediction accuracy of D and H based taper models. Rustagi and Loveless (1990) compared the form-factor based taper model with the variable form taper model by Kozak (1988) and crown ration based taper model by Walters and Hann (1986), and found that it outperformed both when tested on coastal Douglas-fir trees.

FIELD MEASUREMENT OF $H_{.67D}$

If $H_{.67D}$ is an important tree attribute for estimation of the cubic volume and for the prediction of upper-stem diameters, then unless it can be measured on standing trees, it will have little practical use. The $H_{.67D}$ averages one-half of the total tree height and therefore, is in the lower part of the crown and should ordinarily be visible. The parallel bands on the Relaskop provide one way to identify this point and obtain its

Table 2.—Comparative evaluation of equations for predicting V_a, F_a, and log (V_a). Data from 350 felled Douglas-fir trees from five national forests-Region IV, U.S.F.S.

No.	Model	FI	MSD $(ft^3)^2$	SE_v (ft^3)
1	$\hat{V}_a = 0.447 + .001783*D^2*H_a$	0.988	7.350	2.711
2	$\widehat{\log(V_a)} = -2.91947 + 1.75537*\log(D) + 1.26420*\log(H_a)$	0.986	8.561	2.926
3	$\hat{F}_a = 0.340667 + 0.02205*H_a/D$	0.989	6.799	2.607
4	$\hat{V}_a = 0.087 + 0.00071*D^2*H_a + 0.00245*D^2*H_{.67Da}$	0.995	2.840	1.685
5	$\widehat{\log(V_a)} = -2.520863 + 1.933493*\log(D) + 0.435807*\log(H_a) + 0.603348*\log(H_{.67Da})$	0.955	3.023	1.739
6	$\hat{F}_a = 0.154251 + 0.621042*R_{.67Da}$	0.994	3.609	1.900

Published in proceedings of "Interior Douglas-Fir: The Species and Its Management" held February 27 – March 1, 1990 in Spokane, WA, USA.
Compiled and edited by D.M. Baumgartner and J.E. Lotan, Washington State University Extension, Pullman, WA.
(Bulletin Office, WSU, PO Box 645912, Pullman, WA 99164-5912.) MISC0230. 301 pp.

Table 3.—Testing of equations from Table 2, on validation sample of 85 felled Douglas-fir trees from five national forests Region IV, U.S.F.S.

No.	Model	FI	MSD $(ft^3)^2$	SE_v (ft^3)
1	$\hat{V}_a = 0.447 + .001783*D^2*H_a$	0.989	9.249	3.041
2	$\widehat{\log(V_a)} = -2.91947 + 1.75537*\log(D) + 1.26420*\log(H_a)$	0.993	6.233	2.497
3	$\hat{F}_a = 0.340667 + 0.02205*H_a/D$	0.992	6.860	2.619
4	$\hat{V}_a = 0.087 + 0.00071*D^2*H_a + 0.00245*D^2*H_{.67Da}$	0.998	1.343	1.159
5	$\widehat{\log(V_a)} = -2.520863 + 1.933493*\log(D) + 0.435807*\log(H_a) + 0.603348*\log(H_{.67Da})$	0.998	2.077	1.441
6	$\hat{F}_a = 0.154251 + 0.621042*R_{.67Da}$	0.999	0.935	0.967

height at the same time. Another alternative, although cumbersome, would be to use a Wheeler Pentaprism Caliper. Electronic optical devices, currently being developed, which read distances, heights, and diameters may provide a better solution to this problem.

In field trials, the hand-held Relaskop could not be held steady enough to get accurate results. Also, different observations were recorded depending on whether the Relaskop was moved up from below, or down from above. Though not rigorously tested, it appears that a Jake-staff mounted Relaskop should provide satisfactory measurement of $H_{.67D}$ and increased accuracy is likely to result if an arithmatic average of two observations of $H_{.67D}$ is taken: one by moving the Relaskop up from below, and another by moving it down from above. We do not anticipate that any measurement error in $H_{.67D}$ will be serious enough to cancel out the benefits resulting from it inclusion in the upper-stem volume or form-factor prediction models.

LITERATURE CITED

Bruce, D. and D. J. DeMars. 1974. Volume equations for second growth Douglas-fir. USDA Forest Service Res. Note PNW-239. 5 pp.

Burkhart, H. and S. B. Walton. 1985. Incorporating crown ratio into taper equations for loblolly pine trees. For. Sci. 31:478-484.

Demaerschalk, J. P. 1972. Converting volume equations to compatible taper equations. For. Sci. 18:241-245.

Farrar, R. M., Jr. 1985. Crown ratio used as a surrogate for form in a volume equation for natural longleaf pine stems. *In:* Proceedings of the 3rd Biennial Southern Silvicultural Research Conference. USDA Forest Service Gen. Tech. Rep. SO-54, p. 429-435.

Hann, D. W., D. K. Walters and J. A. Scrivani. 1987. Incorporating crown ratio into prediction equations for Douglas-fir stem volume. Can. J. For. Res. 17:17-22.

Honer, T. G. 1964. The use of height and squared diameter ratios for the estimation of cubic foot volume. For. Chronicle 40:324-331.

Kozak, A. 1988. A variable exponent taper equation. Can. J. For. Res. 18:1363-1368.

Newnham, R. M. 1988. A variable form taper function. Information Report PI-X-83. Petwawa Nat. For. Inst., Canada. 31 pp.

Rustagi, K. P. and R. S. Loveless. 1990. Compatible variable-form volume and stem profile equations for Douglas-fir. Submitted to the Can. J. For. Res. for publication. 35 pp.

Schumacher, F. X. and F. S. Hall. 1933. Logarithmic expression of timber tree volume. J. Agric. Res. 47:719-734.

Walters, D. K. and D. W. Hann. 1986. Taper equations for six conifer species in southwest Oregon. Forest Service Lab. Res. Bull. #56. Oregon State Univ. 41 pp.

Authors

John L. Mathis
Regional Mensurationist
Region IV- U.S. Forest Service
Ogden, UT 84401

Krishna P. Rustagi
College of Forest Resources
University of Washington
Seattle, WA 98021

Published in proceedings of "Interior Douglas-Fir: The Species and Its Management" held February 27 – March 1, 1990 in Spokane, WA, USA.
Compiled and edited by D.M. Baumgartner and J.E. Lotan, Washington State University Extension, Pullman, WA.
(Bulletin Office, WSU, PO Box 645912, Pullman, WA 99164-5912.) MISC0230. 301 pp.